BACKPAPER: ALGEBRA III

Please do not consult anyone. You may use the text books or class notes. Please do not use any other resource.

A **ring** would mean a **commutative ring with identity** unless specified otherwise.

(1) (20 points) Write down all the abelian groups of order 600 (upto isomorphism).

(2) (20 points) Prove or disprove. Let $A = \mathbb{Z}[x]/(x^2 - 1)$, $B = \mathbb{Z}[x]/(x^2 + 1)$ be rings.

  (a) A and B are isomorphic as rings.
  (b) A and B are isomorphic as $\mathbb{Z}$-modules.

(3) (20 points) Determine whether the following rings are a PID or a UFD. Justify your answer.
  (a) $A = \mathbb{Q}[x, y, z]/(5x - 1, 3y^2 - 5zx)$
  (b) $B = \mathbb{Z}[x, y, z]/(5x - 1, 3y^2 - 5zx)$
  (c) $C = \mathbb{Z}[x, y, z]/(5x - 1, 3y^2 - 15zx)$
  (d) $D = \mathbb{Z}[x, y, z]/(3x - 1, yz - 1)$

(4) (20 points) Let R be a PID with a unique maximal ideal m and fraction field K. Let $a \in m$ be a nonzero element. Show that $K = R[1/a]$.

(5) (20 points) Let $R = \mathbb{C}[x, y]$ be a polynomial ring in two variables over complex number with fraction field K. Let $M = R \oplus (x, y)R \oplus xR \oplus K$ be an R-module and N be the R-submodule generated by $\{(x, 0, 0, 0), (0, 0, x^3, 0), (0, 0, 0, y^4)\}$. Compute the torsion submodule of M/N and rank of M/N.